TEN YEARS OF PRECISION ELECTROWEAK PHYSICS

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1 Introduction and Brief Historical Perspective

This summer we mark the tenth anniversary of the beginning of a very important period in the study of the Standard Model (SM) [1] and some of its extensions. Namely, in August of 1989, LEP started operations at CERN. Approximately at the same time, SLC and the Mark II detector were switched on at SLAC, and FNAL began the precision studies of the W mass. There have also been important contributions from other great laboratories.

A very attractive feature of this subject and period has been the detailed interplay between theory and experiment. On the experimental side, the accuracy often reaches 0.1% and sometimes it is much better, as in the measurement of the Z mass. On the theoretical side, the study of electroweak corrections to allowed processes, i.e. processes not forbidden in lowest order, has been the basis for the detailed comparisons currently achieved.

Since the path to renormalization has been often reviewed, I will focus on certain aspects and applications of the theory that are closely connected with experiment. This subject is vast and my time limited, so that the topics and references are not intended to be exhaustive. In particular, apologies are due beforehand for the omission of many important contributions.

From the beginning, in order to regularize ultraviolet divergences, most calculations in electroweak physics have been carried out in the dimensional regularization scheme [2]. The application of this approach to regularize infrared divergences and mass singularities was proposed and analyzed somewhat later [3]. In the seventies, radiative corrections to β and muon decays played an important role in the analysis of the universality of the weak interactions and its implications for the phenomenological viability of the SM [4]. The evaluation of the one-loop corrections to $g_\mu - 2$ dates from that period, and there were also a number of important qualitative results, such as the absence of parity and strangeness violating corrections of $O(\alpha)$ to strong interactions [5], the cancellation of ultraviolet divergences in natural relations [6], the discovery that heavy particles do not generally decouple in electroweak corrections and that a heavy top quark gives contributions of $O(G_\mu m_t^2)$ to the ρ parameter [7], and the suppression

of flavor changing neutral currents (FCNC) in $O(G_f \alpha)$ [8]. However, aside from the problem of universality, it was too difficult in the seventies to carry out sufficiently complete calculations of allowed processes that could be compared with experiment at the loop level. One of the main reasons was that, in order to establish connection with experiment, it is generally necessary to evaluate corrections to several processes. This important objective was hampered, partly by the large number of diagrams involved, partly by the complicated nature of the renormalization procedures frequently employed. Since 1980, simple methods to implement the renormalization of the theory have been developed. These simple renormalization frameworks have greatly facilitated the systematic evaluation of the electroweak corrections to important allowed processes, such as muon decay, deep inelastic neutrino-nucleon scattering, neutrino-lepton scattering, atomic parity violation, and e^+e^- annihilation at the Z peak and in the LEP-II domain. In this way, the connection between theory and experiment was finally achieved for a large number of observables. In the early eighties, the main objective was the prediction of m_W and m_Z, using α, G_μ, and the electroweak-mixing parameter $\sin^2 \theta_W$, as inputs. This required the corrections to muon decay [9], and deep inelastic neutrino-nucleon scattering via the neutral [10] and charged currents [11]. The measurement of $\sin^2 \theta_W$ improved with time and by 1987 the calculated vector boson masses were $m_W = 80.2 \pm 1.1\, GeV$ and $m_Z = 91.6 \pm 0.9\, GeV$ [12], with central values within 0.2 GeV and 0.4 GeV from the current ones, respectively.
A major breakthrough took place with the onset of LEP. By the end of August of 1989, m_Z had been measured to within 160 MeV. It became immediately possible to obtain a rather precise value of the $\overline{MS}$ parameter $\sin^2 \hat{\theta}_W(m_Z)$, which in turn helped to verify the consistency with supersymmetric unification [13]. The precision of the m_Z measurement prompted a change in strategy: α, G_μ, and m_Z were adopted as the basic input parameters, a major effort was placed on the study of the observables at the Z resonance, namely the line shape and the various widths and asymmetries measured at LEP and SLC, and there was a great improvement in the comparison between theory and experiment. The main objectives of these studies have been: i) to test the SM at the level of its quantum corrections ii) to predict m_t and constrain the great missing piece, M_H iii) to search for deviations that may signal the presence of new physics beyond the SM.
A good example of the successful interplay between theory and experiment was the m_t prediction and its subsequent measurement. Before 1994-95, the top quark was not observed, but m_t could be inferred indirectly because of its effect on electroweak corrections. In Nov. 1994, a global analysis by the Electroweak Working Group (EWWG) led to the indirect determination $m_t = 178 \pm 11^{+18}_{-19}$ GeV, where the central value corresponds to $M_H = 300$ GeV, the first error is experimental, and the second uncertainty reflects the shift in the central value to $M_H = 65$ GeV and $M_H = 1$ TeV. The present experimental value is $m_t = 174.3 \pm 5.1$ GeV; thus, we see that the indirect determination was in the ball-park. Recent important results include precise

measurements of m_W at CDF, D0, LEP2, and NuTeV.

2 Input Parameters

Three accurate input parameters play a major role in Electroweak Physics.
1) $\alpha = 1/137.03599959(38)(13)$, most precisely derived from $g_e - 2$, with an uncertainty $\delta\alpha = 0.0037$ppm [14].
2) $G_\mu = (1.16637 \pm 0.00001) \times 10^{-5} GeV^{-2}$. It is derived from the muon lifetime with an uncertainty of 9ppm, using the radiative corrections of the V-A Fermi theory. Recently, the two-loop corrections to the muon lifetime have been completed in the approximation of neglecting terms of $O(\alpha^2 (m_e/m_\mu)^2)$ [15]. Using α as expansion parameter, one has:

$$\delta = 1 + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\left[1 + \frac{2\alpha}{3\pi}\ln\left(\frac{m_\mu}{m_e}\right)\right] + 6.701\left(\frac{\alpha}{\pi}\right)^2 + \ldots$$

The contributions of $O(\alpha)$ and $O(\alpha^2 \ln(m_\mu/m_e))$ have been known for a long time [16], while the last term is the new result. The two $O(\alpha^2)$ terms nearly cancel and, including very small $O(\alpha m_e^2/m_\mu^2)$ contributions, one finds:

$$\delta = 1 - 4.1995 \times 10^{-3} + 1.5 \times 10^{-6} + \ldots$$

3) $m_Z = 91.1872 \pm 0.0021 GeV$ with an uncertainty of 23ppm. It turns out that there are subtleties in the theoretical definition of m_Z and the width Γ_Z, associated with issues of gauge invariance and, in the case of photonic and gluonic corrections to $W^\pm$ and quark propagators, with questions involving the convergence of the perturbative expansion in the resonance region [17] . Thus, the study of this region sheds light on the concepts of mass and width of unstable particles!

3 m_W, $\sin^2\theta_W$, On-Shell and $\overline{MS}$ Renormalization Schemes

Other fundamental parameters are m_W , the physical (pole) mass of the W boson and the electroweak mixing parameter. The latter comes in several incarnations, all of them interesting.
$\sin^2\hat{\theta}_W(m_Z) = \hat{s}^2$ is the renormalized parameter in the $\overline{MS}$ renormalization scheme, evaluated at the m_Z scale. A frequently employed version of this method, applied in the early 80's [18], and further developed since 1989 [13, 19, 20, 21] employs $\overline{MS}$ couplings and physical (pole) masses in the electroweak sector. $\hat{s}^2$ is very convenient to discuss physics at the Z peak, and is crucial for GUTs studies.

$\sin^2\theta_W = 1 - m_W^2/m_Z^2 = s^2$ is the renormalized parameter in the on-shell method of renormalization, developed since 1980 [9, 10, 22, 23]. It employs physical parameters such as α, m_W, m_Z, G_μ in the electroweak sector.
Using α, G_μ, and m_Z as inputs, one can evaluate m_W and $\hat{s}^2$, as functions of m_t and M_H. One has the relations:

$$s^2c^2 = \frac{A^2}{m_Z^2(1-\Delta r)} \quad ; \quad \hat{s}^2\hat{c}^2 = \frac{A^2}{m_Z^2(1-\Delta\hat{r})} \quad ; \quad \hat{s}^2 = \frac{A^2}{m_W^2(1-\Delta\hat{r}_W)} \quad ,$$

where $A^2 = \pi\alpha/\sqrt{2}G_\mu$. The corrections Δr [9], $\Delta\hat{r}$ [13, 19], and $\Delta\hat{r}_W$ [19] play an important role in the analysis of electroweak physics, because they link α, G_μ, and m_Z to the precisely determined parameters m_W and $\hat{s}^2$. In particular, as it is clear from the first equation, Δr is a physical observable. Therefore, it can be evaluated in any renormalization scheme.
The $\overline{MS}$ and on-shell definitions of the electroweak mixing angle are related by

$$\hat{s}^2 = s^2\left(1 + \frac{c^2}{s^2}\Delta\hat{\rho}\right) \quad ; \quad \Delta\hat{\rho} = Re\left(\frac{A_{WW}(m_W^2)}{m_W^2} - \frac{A_{ZZ}(m_Z^2)}{m_Z^2\hat{\rho}}\right),$$

where $A_{WW}(m_W^2)$ and $A_{ZZ}(m_Z^2)$ are the W-W and Z-Z self-energies evaluated on their mass-shells, and renormalized in the $\overline{MS}$ scheme at the m_Z scale, and $\hat{\rho} = c^2/\hat{c}^2 = (1-\Delta\hat{\rho})^{-1}$.
The neutral current amplitude is of the form

$$< f\overline{f}|J_\mu^z|0 >= V_f(q^2)\overline{u}_f\gamma_\mu\left[\frac{I_3(1-\gamma_5)}{2} - \hat{k}_f(q^2)\hat{s}^2Q_f\right]v_f,$$

where $\hat{k}_f(q^2)$, its on-shell counterpart $k_f(q^2) = \hat{k}_f(q^2)\hat{s}^2/s^2$, and $V_f(q^2)$ are electroweak form factors, I_3 is the third component of weak isospin, and Q_f is the charge of fermion f. A dominant contribution to $\Delta\hat{\rho}$ can be identified with the top quark contribution to the ρ parameter:

$$\Delta\rho_t = \frac{3G_\mu m_t^2}{8\pi^2\sqrt{2}}(1-0.12) = 8.4\times 10^{-3},$$

where the last factor represents the QCD correction. Similarly, if the neutral current amplitude is parametrized in terms of $G_\mu m_Z^2$, it contains a contribution proportional to $(1-\Delta\rho_t)^{-1}$. However, $\Delta\hat{\rho}$ includes gauge-invariant bosonic contributions and the self-energies in $\Delta\rho_t$ are evaluated at $q^2 = 0$, so that there are significant conceptual and numerical differences between the two corrections.
Another important definition is $\sin^2\theta_{eff}^{lept} = s_{eff}^2$, used by the EWWG to parametrize the data at the Z peak. It is related to the other definitions by $s_{eff}^2 = Re\hat{k}_l(m_Z^2)\,\hat{s}^2 = Rek_l(m_Z^2)\,s^2$ [24], where $\hat{k}_l$ and k_l are the electroweak form factors in the f=lepton

case. By a fortuitous cancellation of electroweak corrections, $\hat{k}_l(m_Z^2)$ is very close to 1 and, for current m_t values, $s_{eff}^2 - \hat{s}^2 \approx 1 \times 10^{-4}$. Writing $\hat{k}_l = 1 + \Delta\hat{k}_l$, and taking into account the smallness of $\Delta\hat{k}_l$, the combination with the expression for $\hat{s}^2\hat{c}^2$ leads to

$$s_{eff}^2 c_{eff}^2 = \frac{A^2}{m_Z^2(1-\Delta r_{eff})} \quad ; \quad \Delta r_{eff} = \Delta\hat{r} + \left(1 - \frac{\hat{s}^2}{\hat{c}^2}\right) Re\Delta\hat{k}_l.$$

As Δr, Δr_{eff} is scale independent, since it is defined in terms of observable quantities. Other interesting renormalization schemes include the formulation presented in Ref. [25]. However, the on-shell and $\overline{MS}$ renormalization schemes remain the most frequently employed, within the SM and beyond. For example, the ZFITTER and BHM programs are based on the on-shell method, while the GAPP [26] and TOPAZ0 codes employ the $\overline{MS}$ formulation.

4 The Running of α. Asymptotic Behavior of Basic Corrections

An important contribution to the basic electroweak corrections is associated with the running of the QED coupling at the m_Z scale: $\alpha(m_Z)/\alpha = 1/(1-\Delta\alpha)$. The contribution of the light quarks (u through b) is evaluated using dispersion relations and $\sigma_{exp}(e^+ + e^- \rightarrow hadrons)$ at low $\sqrt{s}$ and perturbative QCD (PQCD) at large $\sqrt{s}$. A frequently employed value is $\Delta\alpha_h^{(5)} = 0.02804 \pm 0.00065$ [27]. Recently, several "theory driven" calculations claim to sharply reduce the error by extending the application of PQCD to much lower $\sqrt{s}$ values ($\sqrt{s} \approx 1.7 GeV$). An example is $\Delta\alpha_h^{(5)} = 0.02770 \pm 0.00016$ [28]. There is a new calculation that applies PQCD to the Adler function $D(Q^2) = Q^2 \int_{4m_\pi}^{\infty} ds' R(s')/(s'+Q^2)^2$, down to $\sqrt{Q^2} = 2.5 GeV$, where $Q^2 = -s$ is space-like [29]. The authors find $\Delta\alpha_h^{(5)}(-m_Z^2)$, evaluate the difference with $\Delta\alpha_h^{(5)}(m_Z^2)$ using PQCD, and obtain $\Delta\alpha_h^{(5)}(m_Z^2) = 0.027782 \pm 0.000254$. The space-like approach circumvents possible problems associated with time-like thresholds.

The basic corrections have been studied in great detail by several groups. It is instructive to display their asymptotic behaviors for large m_t and M_H:

$$\Delta r \sim -\frac{3\alpha}{16\pi s^4}\frac{m_t^2}{m_Z^2} + \frac{11\alpha}{24\pi s^2} \ln\left(\frac{M_H}{m_Z}\right) + \ldots$$

$$\Delta r_{eff} \approx \Delta\hat{r} \sim -\frac{3\alpha}{16\pi\hat{s}^2\hat{c}^2}\frac{m_t^2}{m_Z^2} + \frac{\alpha}{2\pi\hat{s}^2\hat{c}^2}\left(\frac{5}{6} - \frac{3}{4}\hat{c}^2\right) \ln\left(\frac{M_H}{m_Z}\right) + \ldots$$

These formulae exhibit some of the main qualitative features of the corrections: a quadratic dependence on m_t, enhanced by a relative factor c^2/s^2 in Δr, a logarithmic

dependence on M_H, and opposite signs. The latter leads to a well-known correlation between the m_t and M_H values derived from the electroweak data. Variations of Δr and $\Delta\hat{r}$ induce shifts $\delta m_W/m_W \approx -0.22\delta(\Delta r)$ and $\delta s^2_{eff}/s^2_{eff} \approx 1.53\delta(\Delta\hat{r})$.

5 Evidence for Electroweak Corrections

We discuss two classes of interesting loop contributions.
A) Corrections Beyond the Running of α.
A sensitive argument is to measure Δr [30]. Using the current world average $m_W = 80.394 \pm 0.042 GeV$ [31] , one finds $(\Delta r)_{exp} = 0.03447 \pm 0.00251$, while the contribution to Δr from the running of α is $\Delta\alpha = 0.05954 \pm 0.00065$. Thus, the electroweak correction not associated with the running of α is $(\Delta r)_{exp} - \Delta\alpha = -0.02507 \pm 0.00259$, which differs from zero by 9.7σ! If, instead, one employs the m_W value from the global fit, which includes both direct and indirect information, the evidence for corrections beyond the running of α is close to the 14σ level.
A similar result is obtained by comparing s^2_{eff} and $\sin^2\theta_W$, both of which are physical observables [30]. Their numerical difference arises from electroweak corrections not involving $\Delta\alpha$, and amounts to 0.00879 ± 0.00083, a 10.6 σ effect.
B) Electroweak Bosonic Corrections (EBC).
These include loops involving the bosonic sector, W's, Z, H. They are sub-leading numerically, relative to the fermionic contributions, but very important conceptually. Strong evidence for these corrections can be obtained from Δr_{eff} [32]. Using the current average $s^2_{eff} = 0.23151 \pm 0.00017$, one finds $(\Delta r_{eff})_{exp} = 0.06052 \pm 0.00048$. Subtracting the EBC diagrams (a gauge invariant and finite subset) from the theoretical evaluation, one obtains $(\Delta r_{eff})^{subtr}_{theor} = 0.05106 \pm 0.00083$. The difference is 0.00946 ± 0.00096, a 9.9σ effect!

6 Theoretical Pursuit of the Higgs Boson

With m_t measured, the question of whether and to what extent it is possible to constrain M_H , becomes of considerable interest. One faces the problem that asymptotically the electroweak corrections are $\sim \ln(M_H/m_Z)$, so that precise calculations are necessary. It is therefore important to consider the level of accuracy of the electroweak corrections.
Theorists distinguish two classes of errors: 1) parametric 2) uncertainties due to the truncation of the perturbative series (i.e. un-calculated higher order effects). The first class includes the errors in m_Z, $\Delta\alpha^{(5)}_h$, m_t, s^2_{eff}, m_W, and $\alpha_s(m^2_Z)$. In principle, they can be decreased by improved experiments. The second class is more difficult to estimate. What is the current theoretical situation? Corrections of $O(\alpha)$, $O([\alpha\ln(m_Z/m_f)]^n)$, and $O(\alpha^2\ln(m_Z/m_f))$, where m_f is a generic light-fermion mass,

were analyzed from around 1979 to 1984 [22, 33]. Those of $O(\alpha^2(M_t^2/M_W^2)^2)$ [34], as well as the QCD corrections of $O(\alpha\alpha_s)$ [21, 35] and $O(\alpha\alpha_s^2(M_t^2/M_W^2))$ [36], were studied from the late eighties to the middle nineties.
Of more recent vintage is the analysis of the corrections of $O(\alpha^2 m_t^2/m_W^2)$. Large m_t expansions were employed to evaluate the irreducible contributions of this order to Δr and $\Delta\hat{r}_W$ in the framework of the $\overline{MS}$ renormalization scheme [37]. In order to estimate the theoretical error due to the truncation of the perturbative series, it was very useful to carry out analogous calculations in other schemes, such as the on-shell renormalization framework. It was also important to incorporate these effects in the theoretical evaluation of s_{eff}^2. In order to achieve these goals, the corrections of $O(\alpha^2 m_t^2/m_W^2)$ in the calculation of m_W, s_{eff}^2, and $\hat{s}^2$ were incorporated and compared, as functions of M_H, in three schemes : $\overline{MS}$, and two versions , OSI and OSII, of the on-shell scheme, with two different implementations of the QCD corrections [38]. A large reduction was found in the scheme dependence. The maximal differences among the three calculations, for given M_H, amounted to $\Delta s_{eff}^2 \approx 3\times 10^{-5}$ and $\Delta m_W \approx 2MeV$ while, without the incorporation of the new corrections, the variations were $\approx 2\times 10^{-4}$ and $\approx$ 11 MeV, respectively. Including additional QCD uncertainties, the estimated errors became $\Delta s_{eff}^2 \approx 6\times 10^{-5}$, $\Delta m_W \approx 7MeV$ [39].
The $O(\alpha^2 m_t^2/m_W^2)$ corrections in the calculation of the partial widths $\Gamma_f (f \neq b)$ was studied in Ref. [40], and a partial check of the accuracy of the Heavy Top Expansion was carried out in Ref. [41].
The incorporation of the $O(\alpha^2 m_t^2/m_W^2)$ contributions had a felicitous consequence: it led, for equal inputs, to a significant reduction in the derived value of M_H and its upper bounds. For example, a fit to the data by the EWWG , without inclusion of these effects, led in Aug. 1997 to $M_H < 420GeV$ at 95% CL, while Ref. [39], using the same input values, reported $M_H < 295GeV$, a 30% reduction! The $O(\alpha^2 m_t^2/m_W^2)$ contributions of Refs. [37, 38] have been incorporated for some time in the Erler-Langacker analysis and, more recently, into the ZFITTER and TOPAZ0 codes.

7 Simple Formulae for $\sin^2\theta_{eff}^{lept}$ and m_W

It turns out that simple and accurate formulae for $\sin^2\theta_{eff}^{lept}$ and m_W, as functions of M_H, m_t, $\Delta\alpha_h^5$, and $\alpha_s(m_Z)$, are available in the $\overline{MS}$, OSI, and OSII schemes [39]. For example, in the $\overline{MS}$ framework, one has:

$$\sin^2\theta_{eff}^{lept} = 0.231510 + 0.000523\,\ln\left(\frac{M_H}{100}\right) + 0.00986\left(\frac{\Delta\alpha_h^{(5)}}{0.0280} - 1\right)$$
$$-0.00278\left(\left(\frac{m_t}{175}\right)^2 - 1\right) + 0.00045\left(\frac{\alpha_s(m_Z)}{0.118} - 1\right), \qquad \text{(I)}$$

$$m_W = 80.3827 - 0.0579 \ln\left(\frac{M_H}{100}\right) - 0.008 \ln^2\left(\frac{M_H}{100}\right) - 0.517\left(\frac{\Delta\alpha_h^{(5)}}{0.0280} - 1\right)$$

$$+ 0.543 \left(\left(\frac{m_t}{175}\right)^2 - 1\right) - 0.085 \left(\frac{\alpha_s(m_Z)}{0.118} - 1\right), \qquad \text{(II)}$$

where m_t, M_H, and m_W are expressed in GeV units. These formulae reproduce accurately the detailed numerical results obtained in Ref. [38] in the range $75 \leqq M_H \leqq 350$ GeV, with maximum absolute deviations of $(1.1 - 1.3) \times 10^{-5}$ in the case of Eq.(I), and $(0.8 - 0.9)$ MeV in that of Eq.(II).
As an illustration, using $s^2_{eff} = 0.23151 \pm 0.00017$ [31], $m_t = 174.3 \pm 5.1$ GeV [31], $\Delta\alpha_h^{(5)} = 0.02804 \pm 0.00065$ [27], and $\alpha_s(m_Z) = 0.119 \pm 0.003$ [31] in Eq.(I), one finds $\ln(M_H/100) = -0.077 \pm 0.629$, which corresponds to a central value $M_H^c = 93\, GeV$ and a 95% CL upper bound $M_H^{95} = 260\, GeV$. Inserting this value of $\ln(M_H/100)$ in Eq.(II), one obtains the accurate SM prediction $m_W = 80.381 \pm 0.028 GeV$, to be compared with the current world average $(m_W)_{exp} = 80.394 \pm 0.042 GeV$.
If, instead, one uses $m_W = 80.394 \pm 0.042 GeV$ as input in Eq(II), one finds $M_H^c = 73\, GeV$ and $M_H^{95} = 294\, GeV$. Thus, we see that the m_W measurement already leads to constraints on M_H not far from those derived from s^2_{eff}!

8 Recent values of m_W and $\sin^2\theta^{lept}_{eff}$, and salient results from global fits

Recent values of m_W include the $p - \overline{p}$ average from CDF and D0 : $m_W = 80.448 \pm 0.062\, GeV$ and the LEP-II result $m_W = 80.350 \pm 0.056\, GeV$. The difference between these two measurements is 1.2σ and their average is $m_W = 80.394 \pm 0.042\, GeV$. The NuTeV/CCFR value, whose extraction depends weakly on m_t and M_H, is $m_W = 80.25 \pm 0.11\, GeV$.
In order to discuss s^2_{eff}, it is convenient to recall the parity-mixing amplitude $A_f = 2v_f a_f/(v_f^2 + a_f^2)$, where v_f and a_f are the vector and axial-vector couplings of fermion f with the Z boson at resonance, and the very useful formulae: $A_{FB}^{o,f} = (3/4)\, A_e\, A_f$; $A_{LR} = A_e$; $A_{LR}^{FB}(f) = (3/4) A_f$. This summer, a new value has been reported for the leptonic amplitude A_l at SLD: $A_l(SLD) = 0.1512 \pm 0.0020$, which implies $s^2_{eff} = 0.23099 \pm 0.00026$. The combined LEP and SLD value of A_l is $A_l(LEP + SLD) = 0.1497 \pm 0.0016$, which corresponds to $s^2_{eff} = 0.23119 \pm 0.00020$ (only leptonic amplitudes). There has also been some change in the forward-backward asymmetry $A_{FB}^{o,b}$ in the $b - \overline{b}$ channel. According to a recent analysis [31], the new world-average is $s^2_{eff} = 0.23151 \pm 0.00017$ with χ^2/d.o.f. = 11.9/6. Statistically, this corresponds to a Confidence Level of 6.4%, which is rather low. In particular, the difference between the two most precise values of s^2_{eff}, derived from $A_l(SLD)$ and $A_{FB}^{o,b}$, is 2.9 σ. A varia-

tion of the same magnitude exists between the "leptonic" and "hadronic" averages of s^2_{eff}, involving A_l and A_q, respectively. This can be compared with χ^2/d.o.f. = 7.8/6 at the time of the Vancouver Conference last year, with a statistical confidence level of 26%. Thus, the fit to this crucial parameter is less harmonious at present (summer of 1999) than it was last year.

We now present some salient results from two recent global fits.

The Erler-Langacker fit, which is based on the $\overline{MS}$ scheme, leads to [26] $\hat{s}^2 = 0.23117 \pm 0.00016$, $m_t = 172.9 \pm 4.6 GeV$, $\alpha_s(m_Z) = 0.1192 \pm 0.0028$. They employ a value of $\Delta\alpha_h^{(5)}$ which is adjusted in the fit and correlated with that of $\alpha_s(m_Z)$, and a definition of $\hat{s}^2$ in which the contribution of the top quark is decoupled. In such a case the difference with s^2_{eff} amounts to 2.9×10^{-4} [24], so that $s^2_{eff} = 0.23146 \pm 0.00016$, a fit value somewhat lower than the one derived directly from the various asymmetries measured at LEP and SLC. Their indirect determination of M_H is

$$M_H = 98^{+57}_{-38}\, GeV \qquad ; \qquad M_H^{95} = 235\, GeV.$$

The 95% CL upper bound M_H^{95} takes into account the exclusion constraint from the direct searches of H which, at the time, was $M_H > 95 GeV$. This fit has a χ^2/d.o.f. = 42/37, corresponding to a CL$\approx$ 25%.

A second fit [42], uses the ZFITTER program employed by the EWWG, a code based on the on-shell framework. The value of s^2_{eff} is the one mentioned before, $s^2_{eff} = 0.23151 \pm 0.00017$. Using $\Delta\alpha_h^{(5)} = 0.02804 \pm 0.00065$, it leads to $m_t = 173.2^{+4.7}_{-4.4}\, GeV$, $\alpha_s(m_Z) = 0.1184 \pm 0.0026$, $M_H = 77^{+69}_{-39}\, GeV$, and $M_H^{95} = 215\, GeV$, which does not take into account the exclusion constraint from the direct searches. Instead, employing $\Delta\alpha_h^{(5)} = 0.02784 \pm 0.00026$, one of the recent theory driven calculations, the value of m_t changes by only 0.2 GeV, $\alpha_s(m_Z)$ is unchanged, and $M_H = 90^{+57}_{-37}\, GeV$.

In general, the m_t values from the global fits are smaller by $1 - 2 GeV$ from the direct measurements and, through the $m_t - M_H$ correlation mentioned before, this tends to lower somewhat the M_H value. The new theory driven calculations of $\Delta\alpha_h^{(5)}$ give a larger central value for M_H and a smaller error than the conventional calculations. As a consequence, the upper bound M_H^{95} turns out to be rather close in the two cases. The largest deviations between observables and the values in the Erler-Langacker fit are in $A_{FB}^{o,b} : -2.3\sigma$ (LEP) ; in the atomic parity-violation in Cs, $Q_W(Cs) : 2.3\sigma$; in the leptonic amplitude measured at SLC, $A_l : 1.8\sigma$ (SLC) ; and in the hadronic cross-section at resonance, $\sigma_{had} : 1.7\sigma$ (LEP). (It is important to note that the theoretical error in $Q_W(Cs)$ has been greatly reduced recently [43]). Thus, there are only two observables with deviations larger than 2σ and two additional ones with variations close to 2σ.

However, if one combines $A_{FB}^{o,b}$, $A_l(LEP + SLC)$, and $A_b(SLC)$, one obtains a value for the b-quark amplitude A_b which deviates from the SM model prediction by -2.7σ.

The question has been raised of whether this is due to a statistical fluctuation, or possible new physics coupled to the third generation [44, 45]. In particular, Ref. [45] discusses the implications of the new physics scenario for FCNC. On the other hand, the analysis shows that, if the effect is due to new physics, a substantial, tree-level change in the right-handed $Z - b\bar{b}$ coupling is required.

9 Vacuum Stability and Perturbation Theory Constraints. Supersymmetry

In the very hypothetical scenario in which the SM is valid up to energy scales $\Lambda \sim 10^{19}\, GeV$, one has the theoretical inequality $134\, GeV \lesssim M_H \lesssim 180\, GeV$, where the lower and upper bounds arise from the requirements of vacuum stability [46] and the validity of Perturbation Theory, respectively [47]. These estimates are for $m_t = 175\, GeV$ and $\alpha_s(m_Z) = 0.118$. If, instead, $\Lambda \sim 10\, TeV$, the inequality becomes $85\, GeV \lesssim M_H \lesssim 480\, GeV$.
Over the last several years, supersymmetric scenarios have emerged as leading candidates for physics beyond the SM.
It has been known for a long time that the precision data is compatible with SUSY grand-unification (SUSY GUTs) at $\sim 2 \times 10^{16}\, GeV$! For instance, assuming supersymmetric unification of couplings, and using $\hat{\alpha}(m_Z)$, $\hat{s}^2(m_Z)$ as inputs, one derives $\alpha_s(m_Z) = 0.13 \pm 0.01$, which is consistent with current experimental values [48].
A major prediction of the Minimal Supersymmetric Standard Model (MSSM) is $m_h \lesssim 135 GeV$, where m_h stands for the mass of the lightest CP-even Higgs scalar. Recent diagrammatic calculations of m_h include terms of $O(\alpha\alpha_s)$ and significantly reduce the $\tan\beta$ region that can be probed by the Higgs boson search at LEP-II [49].
In the MSSM, SUSY contributions decouple if the superpartners' masses are much larger than m_Z. In that regime, the fits are of the same general quality as in the case of the SM. If some of them are of $O(m_Z)$, the fits are worse, which leads to constraints in SUSY parameter space. In particular, both non-oblique and oblique corrections are important in that case [50].

10 Constraints on Additional Fermions and Bosons; S,T,U parameters

Erler and Langacker [26] introduce a parameter $\rho_o = c^2/(\hat{c}^2 \hat{\rho}_{SM})$, where $\hat{\rho}_{SM}$ is the SM value of $\hat{\rho}$. It is sensitive to contributions from non-degenerate additional doublets, and non-standard H bosons transforming according to representations other than singlets and doublets. Fitting the data with this additional parameter, they find $\rho_o = 0.9998^{+0.0011}_{-0.0006}$, $95\, GeV < M_H < 211\, GeV$, $m_t = 173.6 \pm 4.9\, GeV$, $\alpha_s(m_Z) =$

0.1194 ± 0.0028, where the lower M_H limit was the direct search bound at the time. This fit is in excellent agreement with the SM prediction $\rho_o = 1$. At the 2σ level one has $\rho_o = 0.9998^{+0.0034}_{-0.0012}$ and $M_H < 1002\, GeV$. Thus, in the presence of possible additional contributions to ρ_o, the constraint on M_H becomes very weak. This analysis implies $\sum_i (C_i/3)(\Delta m_i)^2 \leqslant (100 GeV)^2$ at 95% CL, where the sum is over additional fermion and scalar doublets and $C_i = 1(3)$ for color singlets (triplets). The bound is sharper for non-degenerate squark and slepton doublets, namely $(69\, GeV)^2$, on account of the strong correlation between ρ_o and the restricted SUSY value of m_h.
The S,T, and U parameters are very useful to analyze possible new physics contributions that reside in the self-energies (also called oblique corrections) and which involve generic masses $m_i >> m_Z$ [51]. One has

$$\hat{\alpha}(m_Z)\, T \equiv \frac{A^{new}_{WW}(0)}{m_W^2} - \frac{A^{new}_{ZZ}(0)}{m_Z^2} \quad ; \quad \frac{\hat{\alpha}(m_Z)}{4\hat{s}^2\hat{c}^2}\, S \equiv \frac{A^{new}_{ZZ}(m_Z^2) - A^{new}_{ZZ}(0)}{m_Z^2}$$

$$\frac{\hat{\alpha}(m_Z)}{4\hat{s}^2}\, (S+U) \equiv \frac{A^{new}_{WW}(m_W^2) - A^{new}_{WW}(0)}{m_W^2},$$

where the superscript new indicates that only new physics contributions are included. They are part of the new physics contributions to the basic corrections [52]. In fact, we have:

$$\delta(\Delta\hat{r}) \approx \delta(\Delta r_{eff}) = \frac{\alpha}{4\hat{s}^2\hat{c}^2}\, S - \alpha\, T \quad ; \quad \delta(\Delta r_W) = \frac{\alpha}{4\hat{s}^2}\, (S+U)$$

$$\delta(\Delta r) = \frac{\alpha}{2\hat{s}^2}\, S - \frac{\alpha}{4\hat{s}^2}\left(\frac{c^2}{s^2} - 1\right) U - \alpha \frac{c^2}{s^2} T.$$

For contributions involving masses $m_i >> m_Z$, the S and S+U parameters are approximately proportional to the wavefunction renormalizations of the Z and W bosons, respectively. Heavy non-degenerate additional doublets contribute positively to $\hat{\alpha}\, T = \rho_o - 1$, and to a lesser extent to U. The S parameter, instead, is sensitive to heavy degenerate chiral fermions, with a contribution $S = 2/3\pi$ per generation. As loops involving the Higgs boson affect mostly the self-energies, the S, T, and U parameters cannot be fitted simultaneously with M_H. Therefore, a value of M_H is usually assumed. A recent fit [26] gives $S = -0.07 \pm 0.11(-0.09), T = -0.10 \pm 0.14(+0.09), U = 0.11 \pm 0.15(0.01)$. The central values assume $M_H = 100\, GeV$, while the values in parentheses indicate the change corresponding to $M_H = 300\, GeV$. These determinations are consistent with the SM values $S = T = U = 0$. The result for S indicates that negative contributions to this parameter can remove the SM constraint on M_H. For $M_H = 600\, GeV$ and $S > 0$, as is appropriate in the simplest technicolor models, $S \leqslant 0.09$, which rules out models of this type involving many techni-doublets. These bounds can be evaded in models of walking technicolor, where S may be small or negative. For $T = U = 0$, a fit to the S parameter excludes an additional generation

of degenerate chiral fermions at the 99.6% CL. Allowing $T = 0.18 \pm 0.08$, the CL becomes 97%. This exclusion of an additional generation is generally consistent with the number of light neutrinos $N_\nu = 2.985 \pm 0.008$, obtained from the invisible width of the Z boson. An alternative formulation to S,T, and U, involves the ϵ_i parameters, defined in terms of physical observables, m_W, Γ_l, $A_{FB}^{o,l}$, and $\Gamma_{b\bar{b}}$ [53].

11 Additional Z' and excited $W^{\pm *}$ Bosons; Trilinear Gauge Couplings

Additional Z' bosons appear naturally in many Grand Unified Theories (GUTs) and superstring-inspired models. Frequently discussed examples are Z_χ of $SO(10) \to SU(5) \times U(1)_\chi$, Z_ψ of $E_6 \to SO(10) \times U(1)_\psi$, Z_η of $E_6 \to SU(5) \times U(1)_\eta$, Z_{LR} of $SU(2)_L \times SU(2)_R \times U(1)$. The Z_η is a linear combination of Z_χ and Z_ψ. In previous analyses [26], it was found that the mixing angles between Z and Z' are severely constrained and that the 95% CL lower bounds were several hundred GeV, except for Z_ψ, which contains only axial vector couplings to ordinary fermions. Furthermore, in certain models in which the Higgs U(1)' quantum numbers are specified, the lower bounds were pushed into the TeV region. Very recently [54] it has been argued that, because of the deviations in σ_{had} (1.7σ) and $Q_W(Cs)$ (2.3σ) relative to the SM predictions, a better fit to the data can be actually achieved by assuming the existence of an additional Z'. In the Z_χ scenario, these authors find $M_{Z'} = 812^{+339}_{-152}\, GeV$, $M_H = 145^{+103}_{-61}\, GeV$, a mixing angle of $O(10^{-3})$ and $\chi^2/d.o.f. = 35/35$. In the Z_{LR} case, the predictions are $M_{Z'} = 781^{+362}_{-241}\, GeV$, and $M_H = 165^{+155}_{-91}\, GeV$, while in a class of E_6 models the results are $M_{Z'} = 287^{+673}_{-101}\, GeV$ and $M_H = 101^{+57}_{-39}\, GeV$. The order of magnitude of the mixing angles and the confidence level in the last two cases are similar to those found in the Z_χ analysis.
Another interesting new physics scenario involves excited $W^{\pm *}$ bosons that may arise as Kaluza-Klein excitations in certain theories with additional compact dimensions, or in models with composite gauge bosons. Assuming couplings identical to those of the SM W, direct searches at the Tevatron lead to $m_{W*} > 720\, GeV$ at 95% CL. These excited W^* contribute to muon decay, so that the amplitude for this process is proportional to $< m_W^2 >^{-1} = (m_W^2)^{-1} + (g_2^*/g_2)^2\, (m_W^{*2})^{-1} + \ldots$. Assuming $M_H \lesssim 200\, GeV$, and using the theoretical expression involving $\Delta\hat{r}$, one can determine $< m_W^2 >^{-1}$ from muon decay [44]. Comparing this result with the measured value $(m_W)_{exp}$, one finds $m_{W*} > 2.9(g_2^*/g_2)\, TeV$ at 95% CL. This suggests that, in these theories, the radius of the extra dimension is $R \approx 1/m_{W*} < 10^{-17}(g_2/g_2^*)$ cm. Of course, the bound is significantly relaxed if $g_2^* << g_2$.
A very important verification of the SM involves the measurement of the trilinear couplings of $W^\pm$ with the photon and Z boson. Under a number of theoretical assumptions, the deviations from the SM couplings can be parametrized in terms of

three quantities, $\Delta\kappa_\gamma$, λ_γ, and Δg_1^Z. The first two contribute to the magnetic and quadrupole moments of $W^\pm$ according to the relations: $\mu_W = (e/2m_W)(1+\kappa_\gamma+\lambda_\gamma)$ and $Q_W = -(e/2m_W)(\kappa_\gamma - \lambda_\gamma)$. The SM predictions are $\kappa_\gamma = 1$, and $\lambda_\gamma = 0$, so that one defines $\Delta\kappa_\gamma = \kappa_\gamma - 1$. Setting two of the parameters to zero and fitting the third, recent measurements at LEP-II [26] give $\Delta\kappa_\gamma = 0.038^{+0.079}_{-0.075}$, $\lambda_\gamma = -0.037^{+0.035}_{-0.036}$, $\Delta g_1^Z = -0.010 \pm 0.033$, which are consistent with the null prediction of the SM.

12 Electroweak Baryogenesis

The current consensus is that it is not possible to explain baryogenesis within the framework of the SM. In the usual scenario, involving the formation of expanding bubbles with a broken phase, necessary conditions are sufficiently strong CP violation, a first-order electroweak phase transition, and the inequality $v(T_c) > T_c$ where T_c is the critical temperature for the transition and $v(T_c)$ the electroweak scale at that temperature. These conditions are met only for values of M_H substantially smaller than the current lower bounds. Thus, in order to explain baryogenesis in the current scenario, one must invoke physics beyond the SM. In the MSSM, the conditions can be met if $m_h \lesssim 105\,GeV$ for $m_{\tilde{t}_L} \lesssim 1\,TeV$, or $m_h \lesssim 115\,GeV$ if $m_{\tilde{t}_L}$ reaches the few TeV region [55]. In both cases the right-handed stop mass is restricted to lie in the range $100\,GeV \lesssim m_{\tilde{t}_R} \lesssim m_t$. Thus, in the MSSM there is a "window of opportunity". On the other hand, it is worth noting that the current lower limit on M_H has recently reached the $105\,GeV$ level!

13 $g_\mu - 2$

The current SM prediction is $a_\mu^{SM} = 116591596(67) \times 10^{-11}$, while the experimental value is $a_\mu^{exp} = 116592350(730) \times 10^{-11}$ [14]. Thus, the difference is $a_\mu^{exp} - a_\mu^{SM} = (754 \pm 733) \times 10^{-11}$ At the two-loop level, the electroweak contribution amounts to $a_\mu^{EW} = 151(4) \times 10^{-11}$, and the hadronic part of the QED contribution is $a_\mu^{had} = 6739(67) \times 10^{-11}$. The present goal is to reduce errors to the 40×10^{-11} level. a_μ is sensitive to several types of new physics [14]. Of particular interest is the SUSY contribution $a_\mu^{SUSY} \approx 140 \times 10^{-11}(100\,GeV/\tilde{m})^2 \tan\beta$, arising from $\tilde{\nu}\tilde{\chi}^-$ and $\tilde{\mu}\tilde{\chi}^o$ loops, which will probe the large $\tan\beta$ scenario.

14 Conclusions

i) At present levels of accuracy, the SM describes very well the results of a large number of experiments ranging from the atomic scale to about $200\,GeV$.
ii) Simple renormalization frameworks have been developed since ≈ 1980 and have

been applied systematically to study the electroweak corrections to a large variety of allowed processes. They play an important role in current analyses.
iii) Two-loop corrections to the muon lifetime in the V-A Fermi theory have been completed.
iv) The theoretical analysis of the resonance propagators (line shape) sheds light on the concepts of mass and width of unstable particles. In particular, in the perturbative regime, the conventional expressions are valid in next to leading order, but not beyond.
v) Theory-driven calculations of $\Delta\alpha_h^{(5)}$ claim to reduce the error by a factor ≈ 4. However, the 95% upper bound M_H^{95} is rather insensitive to the change, because of a partial cancellation between the reduced error and a shift to larger M_H central values.
vi) The evidence for electroweak corrections beyond the running of α and for electroweak bosonic corrections has become very sharp: $\gtrsim 10\,\sigma$!
vii) Trilinear gauge couplings agree very well with SM predictions.
viii) With the discovery of the top quark, efforts to constrain M_H have become increasingly interesting.
ix) In particular, the incorporation of the $O(\alpha^2 m_t^2/m_W^2)$ corrections has significantly decreased the scheme dependence (theoretical error) of the calculations and reduced M_H^{95} by $\approx 30\%$!
x) Simple and accurate formulae for $\sin^2\theta_{eff}^{lept}$ and m_W are available.
xi) A major task for the future is the full evaluation of two-loop contributions to $\Delta r, \Delta\hat{r}, \Delta\hat{k}, \ldots$. This would be crucial if $\delta s_{eff}^2 \to 0.01\%$ at the NLC!
xii) The present Erler-Langacker global fit to the electroweak data leads to $M_H^{95} = 235\,GeV$, taking into account the constraint from the direct searches of H. It has $\chi^2/d.o.f. = 42/37$. Only two observables deviate from the SM predictions by more than 2σ, with two others at the $\approx 1.8\sigma$ level. But, the combined A_b differs by -2.7σ. It has been argued that this may be due to new physics coupled to third generation, in which case new sizeable, tree-level contributions to $g_R^{b\bar{b}}$ are required.
xiii) Supersymmetry: the precision data is consistent with grand-unification of couplings at $\approx 2\times 10^{16}\,GeV$. A major prediction of the MSSM is $m_h \lesssim 135\,GeV$ (and less for small $\tan\beta$). The current consensus is that Electroweak Baryogenesis is not feasible in the SM, but there remains a "window of opportunity" in the MSSM: $m_h \lesssim 105 - 115\,GeV$.
xiv) There are sharp constraints for non-decoupling new physics: sequential generations, simplest QCD-like Technicolor models are disfavored with high probability.
xv) Previous analyses led to lower bounds for masses of additional Z' bosons and sharp constraints on mixing angles. However, it has been recently argued that, because of the current deviations in σ_{had} and $Q_W(Cs)$, a better fit to the precision data can be actually achieved by postulating the presence of an additional Z'. There are also interesting new bounds on excited $W^{\pm *}$ bosons that may arise in certain theories with additional dimensions or in composite models.

xvi) $g_\mu - 2$ is sensitive to SUSY contributions in the large $\tan\beta$ scenario.
xvii) Notwithstanding the current successes of the SM, there remains a plethora of unsolved, fundamental problems: the precise mechanism of symmetry breaking, the explanation of the mass spectrum and the number of generations, the unification with gravity, baryogenesis, the detailed understanding of CP violation, and probably others that will unfold as we continue our quest.

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